Test (HL) – Probability Distributions

Syllabus content: ▪ discrete and random variables and their probability distributions ▪ expected value (mean), median, variance and standard deviation ▪ binomial distribution ▪ normal distribution

total marks on test: **57**

Part I: **No** calculator – questions 1, 2, 3, 4 (a & b)

1. A discrete random variable X has the following distribution table.

x	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{2}{9}$	k	$\frac{1}{3}$	$\frac{1}{9}$

(a) Find the value of k. [2 marks]

(b) Find $\mathrm{E}(x)$. [2 marks]

(c) Find the standard deviation of X. [3 marks]

2. The continuous random variable X has probability density function f defined by

$$f(x) = \begin{cases} ax + b, & 0 \le x \le \frac{3}{2} \\ 0, & \text{otherwise} \end{cases} \quad \text{where } a, b \in \mathbb{R}$$

Given that $\mathrm{E}(x) = \frac{5}{8}$, find the value of a and the value of b. [6 marks]

3. A random variable X has probability density function, $f(x)$, where

$$f(x) = \begin{cases} \sin\left(\frac{x}{2}\right), & 0 \le x \le p \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that $p = \frac{2\pi}{3}$. [3 marks]

(b) Find the **exact** value of the expected value of X. [6 marks]

4. A multiple choice test has 20 questions and each question has four answers from which to choose. Each question has only one correct answer. Let X be the number of correct answers chosen if each answer is randomly guessed.

(a) Briefly explain why X can be modelled by a binomial distribution. [2 marks]

(b) Find the mean and standard deviation of X. [4 marks]

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Part II: calculator allowed – questions 4 (c & d), 5, 6, 7 ,8

[Q.4 continued with calculator allowed]

4. (c) With random guessing on the test, find the probability that a student gets exactly three correct answers. [3 marks]

(d) With random guessing on the test, find the probability that a student gets at least three correct answers. [3 marks]

5. Quentin trains his dog Rusty to run an obstacle course. Each day Rusty runs the course once and Quentin records how long it takes Rusty to run the course. The time it takes Rusty to run the course can be modelled by a normal distribution with a mean of 92 seconds and a standard deviation of 10 seconds. Quentin gives Rusty a treat if he runs the course in less than 80 seconds.

(a) Find the probability that Rusty gets a treat for running the course next Monday. [2 marks]

(b) Find the probability that Rusty gets a treat at least twice during the next week (Monday to Sunday). [4 marks]

6. The masses of a toy truck made in a certain factory are normally distributed with mean μ and standard deviation σ. 5% of the trucks have a mass greater than 85 grams and 10% have a mass less than 25 grams.

(a) Find the values of μ and σ. [5 marks]

(b) 75% of the masses are between the values of $\mu - k$ grams and $\mu + k$ grams. Find the value of *k*. [3 marks]

7. A random variable X has the following probability density function*:*

$$f(x) = \begin{cases} e^{2x}, & 0 \le x \le a \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the **exact** value of *a*. [4 marks]

(b) Find the **approximate** value of the median of *X*. [2 marks]

(c) Three values of this random variable are chosen at random. What is the probability that at least one of the values is larger than $\frac{1}{4}$? [3 marks]